

A Nontrivial Field Theoretical Model

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We show that when a model, which is equivalent to the Gürsey model classically, is gauged with a $SU(N)$ field, we get a nontrivial field theory.

I. INTRODUCTION

To write a field theoretical model which has a nonzero values for the coupling constants at zeroes of the beta function of the renormalization group is an endeavor which is still continuing in particle physics. The ϕ^4 theory is a "laboratory" where different methods in quantum field theory are first applied. After it was shown that this model went a trivial theory when the cut-off was removed, [1, 2], it was clear that analyzing the terms in the perturbation series was not sufficient to decide whether one had a truly interacting theory. Work in this field was also given by Wilson and others, [3, 4]. Renormalization group methods, first introduced by Wilson for this purpose [5], are the most commonly used technique in studying whether one has a trivial theory or not.

Since a nontrivial fixed point is not yet found for QCD, there are attempts to study alternative models for this purpose. A very popular model is the Nambu-Jona-Lasinio model, hereafter NJL, [6]. This model is written in terms of spinor fields only, and is used as an effective theory extensively in high energy physics [7, 8]. The NJL model was also shown to be trivial [9, 10]. Recent attempts to gauge this model to obtain a nontrivial theory are given in references [11, 12, 13, 14, 15]. Both functional and diagram summing methods were used in these papers. Exact renormalization group methods proposed by Wilson and Polchinski, [5, 16], are often employed for this purpose. A very recent paper on this method is given by Sonoda [17].

Another model, which uses only spinors is the Gürsey model [18]. We have worked on different forms of the Gürsey model [19, 20, 21]. Our starting point was both our earlier work, [22, 23, 24, 25, 26], where Gürsey model lagrangian was attempted to be written in a polynomial form, and recent work, [8, 27, 28, 29, 30], which suggested that the gauged form of the NJL model can be interpreted as a nontrivial theory. In [19] we reinterpreted our earlier work, [22], and showed that rather than finding a trivial theory, as claimed in [25], we ended up in a model where composite particles took part in physical processes. The constituent fields, however, did not interact with each other when perturbation theory was applied to the model, as already shown in [25]. In [21], we showed that, when this model is coupled to a constituent $U(1)$ gauge field, we were mimicking a gauge Higgs-Yukawa system, which had the known problems of the Landau pole, with all of its connotations of triviality.

The essential point of our earlier work was the fact that the propagator of the composite scalar field was equal to $\frac{\epsilon}{p^2}$. Since ϵ goes to zero as the cut off removed, many of the diagrams, where the scalar field propagator takes part as an internal line, become convergent. We could show that there was no breaking of the chiral symmetry, thus no mass generation, for the fermion fields in our model in higher orders of perturbation theory.

Here we will study our original model [19], coupled to a $SU(N)$ gauge field and use solely renormalization group techniques. We start with the description of our starting model without the gauge field [19]. Then we derive the renormalization group equations (RGEs) in one loop, and try to derive the criteria for obtaining non-trivial fixed points for the coupling constants of the theory. Here we closely follow the line of discussion followed in our reference [11]. In our model, however, there is a composite scalar field with a propagator completely different from a constituent scalar field used in this reference. This gives rise to RGE's in our case which are different from those given by Harada et al. Since our starting models are different the motivation of our work is different from that of this reference. We show that the renormalization group equations point to the non-triviality of the model when it is coupled to an $SU(N)$ gauge field. We end up with a few remarks in the last section.

II. THE ORIGINAL MODEL

Our initial model is given by the Lagrangian

$$L = i\bar{\psi}\not{\partial}\psi + g\bar{\psi}\psi\phi + \xi(g\bar{\psi}\psi - a\phi^3). \quad (1)$$

Here the only terms with kinetic part are the spinors. ξ is a Lagrange multiplier field, ϕ is a scalar field with no kinetic part, g and a are coupling constants. This expression contains two constraint equations, obtained from writing the

Euler-Lagrange equations for the ξ and ϕ fields. Hence, it should be quantized by using the Dirac constraint analysis as performed in reference [19].

The Lagrangian given above is just an attempt in writing the original Gürsey Lagrangian

$$L = i\bar{\psi}\not{\partial}\psi + g'(\bar{\psi}\psi)^{4/3}, \quad (2)$$

in a polynomial form.

We already showed how the γ^5 invariance of the Gürsey Lagrangian, which prevents the fermion field from acquiring a finite mass in higher orders, is retained in our model, and the fact that our model is equivalent to the original Gürsey model only classically in [19].

To quantize the latter system consistently we proceed via the path integral method. This procedure is carried out in reference [19]. At the end of these calculations we find out that we can write the constrained lagrangian given in equation (1) as

$$L' = i\bar{\psi}[\not{\partial} + ig\Phi]\psi - \frac{a}{4}(\Phi^4 + 2\Phi^3\Xi - 2\Phi\Xi^3 - \Xi^4) + \frac{i}{4}c^*(\Phi^2 + 2\Phi\Xi + \Xi^2)c, \quad (3)$$

where the effective lagrangian is expressed in terms of scalar fields Φ , and Ξ , ghost fields c , c^* and spinor fields only. The fermion propagator is the usual Dirac propagator in lowest order, as can be seen from the lagrangian. After integrating over the fermion fields in the path integral, we obtain the effective action. The second derivative of the effective action with respect to the Φ field gives us the induced inverse propagator for the Φ field, with the infinite part given as

$$Inf \left[\frac{ig^2}{(2\pi)^4} \int \frac{d^4p}{p(p + \not{q})} \right] = \frac{g^2 q^2}{4\pi\epsilon} \quad (4)$$

Here dimensional regularization is used for the momentum integral and $\epsilon = 4 - n$. We see that the Φ field propagates as a massless field.

When we study the propagators for the other fields, we see that no linear or quadratic term in Ξ exists, so the one loop contribution to the Ξ propagator is absent. Similarly the mixed derivatives of the effective action with respect to Ξ and Φ are zero at one loop, so no mixing between these two fields occurs. We can also set the propagators of the ghost fields to zero, since they give no contribution in the one loop approximation. The higher loop contributions are absent for these fields.

In reference [19] we also studied the contributions to the fermion propagator at higher orders and we found, by studying the Dyson-Schwinger equations for the two point function, that there were no new contributions. We had at least one phase where the mass of the spinor field was zero.

III. RENORMALIZATION GROUP EQUATIONS

Here we couple an $SU(N_C)$ gauge field to the model. We start with the effective lagrangian given as

$$L'' = -\frac{1}{4}Tr[F_{\mu\nu}F^{\mu\nu}] - \frac{a}{4}\Phi^4 + \sum_{i=1}^{N_f} \bar{\psi}_i i\not{D}\psi_i - \sum_{i=1}^{N_f} g\Phi\bar{\psi}_i\psi_i. \quad (5)$$

Here N_f is the number of flavors. The gauge field belongs to the adjoint representation of the color group $SU(N_C)$ where D_μ is the color covariant derivative. g, a, e are the Yukawa, quartic scalar and gauge coupling constants, respectively. We take N_f in the same order as N_C .

In the one loop approximation, the renormalization group equations read as

$$16\pi^2 \frac{d}{dt}e(t) = -be^3(t), \quad (6)$$

$$16\pi^2 \frac{d}{dt}g(t) = -cg(t)e^2(t), \quad (7)$$

$$16\pi^2 \frac{d}{dt}a(t) = -ug^4(t), \quad (8)$$

where b, c and u are positive constants given as

$$b = \frac{11N_C - 4T(R)N_f}{3}, \quad c = 6C_2(R), \quad u = 8N_fN_C. \quad (9)$$

Here $C_2(R)$ is a second Casimir, $C_2(R) = \frac{(N_C^2 - 1)}{2N_C}$ and R is the fundamental representation with $T(R) = \frac{1}{2}$. We take μ_0 as a reference scale at low energies, $t = \ln(\mu/\mu_0)$, where μ is the renormalization point.

In the RGE we see that the diagrams, where scalar propagators take part, are down by powers of ϵ . Hence we do not have contributions proportional to $a^2(t)$, $g^3(t)$ and $a(t)g^2(t)$, as one would have in the gauge-Higgs-Yukawa system as described in the work of [11].

A. Solutions of the RGE's

The solution for the first RG equation (6) can be obtained easily as

$$e^2(t) = e_0^2 \left(1 + \frac{b\alpha_0}{2\pi} t \right)^{-1} \quad (10)$$

where $\alpha_0 = \frac{e_0^2}{4\pi}$. Define

$$\eta(t) \equiv \frac{\alpha(t)}{\alpha_0} \equiv \frac{e^2(t)}{e_0^2} \quad (11)$$

where $e_0 = e(t=0)$ which is the initial value at the reference scale μ_0 . For the solution of the second RG equation (7) we can define a RG invariant $H(t)$ as

$$H(t) = (c - b)\eta^{-1+\frac{c}{b}}(t) \frac{e^2(t)}{g^2(t)}. \quad (12)$$

Since $H(t)$ is a constant, we call it H_0 . Then, the solution of the gauge coupling constant can be written as

$$g^2(t) = \frac{(c - b)e_0^2}{H_0} \eta^{\frac{c}{b}}(t). \quad (13)$$

The solution of the last RG equation (8) can be defined by another RG invariant $K(t)$, given as

$$K(t) = -u\eta^{-1+\frac{2c}{b}}(t) \left[1 - \frac{2(2c - b)}{u} \frac{a(t)}{g^2(t)} \frac{e^2(t)}{g^2(t)} \right]. \quad (14)$$

We can then write

$$a(t) = \frac{u}{2(2c - b)} \frac{g^2(t)}{e^2(t)} g^2(t) \left[1 + \frac{K_0}{u} \eta^{1-\frac{2c}{b}}(t) \right]. \quad (15)$$

Here K_0 is the value of the RG invariant. We can rewrite equation (15) as

$$a(t) = \frac{u(c - b)^2 e_0^2}{2H_0^2(2c - b)} \left[\eta^{-1+\frac{2c}{b}}(t) + \frac{K_0}{u} \right]. \quad (16)$$

When we check the ultraviolet limit now, we find

$$\eta(t \rightarrow \infty) \rightarrow +0, \quad b > 0; \quad (17)$$

$$\eta^{\frac{c}{b}}(t \rightarrow \infty) \rightarrow +0, \quad c, b > 0; \quad (18)$$

and

$$\eta^{-1+\frac{2c}{b}}(t \rightarrow \infty) \rightarrow \begin{cases} +0, & 2c > b; \\ +0, & 2c > b > c; \\ +\infty, & b > 2c. \end{cases} \quad (19)$$

We see that the constants H_0 and K_0 play important roles on the behavior of solutions of coupling equations (10),(13),(16). For $c > b$, H_0 should be positive; for $c < b$, H_0 should be negative to have the Yukawa coupling take a real value. This is necessary to have a unitary theory. Also for a region $c < b < 2c$, with $H_0 < 0$, the unitarity condition is satisfied for all coupling constants. $K_0 \geq 0$ condition is also needed for stability of the vacuum. If $K_0 < 0$, we get $a(t \rightarrow \infty) < 0$, which raises the problem of the vacuum instability. Next we study the different limits our parameters can take:

1. $b \rightarrow +0$ limit case for finite t

We find

$$\begin{aligned} e^2(t) &= e_0^2, \\ g^2(t) &= \frac{ce_0^2}{H_0} \exp(-\frac{\alpha}{\alpha_c} t), \\ a(t) &= \frac{uce_0^2}{4H_0^2} \left[\exp(-\frac{2\alpha}{\alpha_c} t) + \frac{K_0}{u} \right]. \end{aligned} \quad (20)$$

Here $\frac{c}{2\pi} = \frac{1}{\alpha_c}$ and $\alpha_0 = \alpha$. This means that when we set the b term to zero, the Yukawa running coupling constant decreases exponentially to zero. For this limit the gauge and the quadratic coupling constants go just to a constant.

2. $c \rightarrow b$ limit case for finite t

If c approaches b , the limit depends on the value of H_0 . If H_0 is non zero, $g^2(t)$ goes to zero. If H_0 goes to zero as a constant times $c - b$, i.e. $H_0 = \frac{c-b}{H_1}$, we find that $g^2(t)$ and $a(t)$ are both proportional to $e^2(t)$ as follows

$$g^2(t) = H_1 e^2(t), \quad H_1 > 0; \quad (21)$$

$$a(t) = \frac{ue_0^2 H_1^2}{2b} \left[\eta(t) + \frac{K_0}{u} \right]. \quad (22)$$

3. $2c \rightarrow b$ limit case for finite t

When $2c$ approaches b , the behavior of $a(t)$ changes. If we set $\frac{K_0}{u} = -1 + \frac{2c-b}{b} K_1$, then $a(t)$ goes as $\ln \eta(t)$

$$a(t) = \frac{ube_0^2}{8H_0^2} \left[K_1 + \ln \eta(t) \right]. \quad (23)$$

This behaviour is not allowed since $a(t)$ diverges as $t \rightarrow +\infty$.

IV. NON-TRIVIALITY OF THE SYSTEM

In this section we use the preceding results to investigate the non-triviality of the system with several criteria such as:

All the running coupling constants:

- should not diverge at finite $t > 0$ (no Landau poles);
- should not vanish identically;
- should not violate the consistency of the theory such as unitarity and/or vacuum stability.

Since the composite scalar field is the novel feature of our model, we will not consider the case when the scalar field is completely decoupled from the theory.

A. Fixed Point Solution

We derive the expressions given below from the RGE equations.

$$8\pi^2 \frac{d}{dt} \left[\frac{g^2(t)}{e^2(t)} \right] = (b - c) \left[\frac{g^2(t)}{e^2(t)} \right] e^2(t), \quad (24)$$

$$8\pi^2 \frac{d}{dt} \left[\frac{e^2(t)}{g^2(t)} \frac{a(t)}{g^2(t)} \right] = (2c - b) \left[\frac{e^2(t)}{g^2(t)} \frac{a(t)}{g^2(t)} - \frac{u}{2(2c - b)} \right] e^2(t). \quad (25)$$

For the fixed point solution, b equals c in equation (24). For this value, there is a single solution which satisfies both equations (24) and (25). This solution is given as,

$$\frac{e^2(t)}{g^2(t)} = \frac{1}{H_1}, \quad (26)$$

where H_1 is a constant, and

$$\frac{a(t)}{g^2(t)} = \frac{uH_1}{2c}. \quad (27)$$

If we take $H_0 = H_1(c - b)$ approaching zero as c approaches to b , while $K_0 = 0$ in equation (16), then we find

$$g^2(t) = H_1 e^2(t), \quad (28)$$

$$a(t) = \frac{uH_1}{2c} g^2(t). \quad (29)$$

Since $\frac{g^2(t)}{e^2(t)}$ and $\frac{a(t)}{g^2(t)}$ are constants, the behavior of the Yukawa and quartic scalar couplings are completely determined by the gauge coupling. This corresponds to "coupling constant reduction" in the sense of Kubo, Sibold and Zimmerman [31]. In the context of the RGE, it corresponds to the Pendleton-Ross (PR) fixed point [32].

B. Yukawa Coupling

As seen from the previous sections the behavior of the Yukawa coupling depends on whether $c > b$ or $c < b$. The point where $c = b$ needs a special care. Moreover the sign of the H_0 is important.

1. $c > b$ case

In this case H_0 should not equal to zero. Then we find in the UV limits

$$g^2(t \rightarrow \infty) \rightarrow \begin{cases} +0, & H_0 > 0; \\ -0, & H_0 < 0. \end{cases} \quad (30)$$

So the Yukawa coupling is asymptotically free. As it is seen, the sign of the RG invariant is important. It should be chosen positive not to cause the violation of stability of the vacuum.

In Figure 1 we plot g^2 vs. e^2 for $c = 8, b = 7$. Both coupling constants approach the origin as t goes to infinity. Thus, our model fulfills the condition required by the asymptotic freedom criterion.

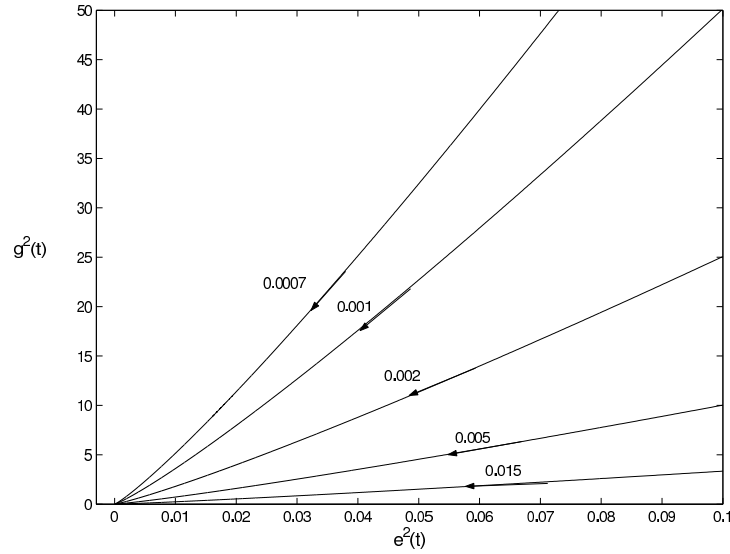


FIG. 1: Plot of $g^2(t)$ vs. $e^2(t)$ for different values of H_0 . The arrows denote the flow directions toward the UV region.

2. $c < b$ case

In this case with a non zero value of H_0

$$g^2(t \rightarrow \infty) \rightarrow \begin{cases} -0, & H_0 > 0; \\ +0, & H_0 < 0. \end{cases} \quad (31)$$

For $H_0 < 0$, our system satisfies the asymptotic freedom condition. Our system does not have a Landau pole. In this respect it differs from the gHY system [11]. As shown below, there is a restriction on the value of b in this case.

3. $c = b$ case

This is the fixed point solution analyzed above.

$$g^2(t) = H_1 e^2(t). \quad (32)$$

C. Quartic Scalar Coupling

$a(t)$ can be analyzed with four non trivial limits of the Yukawa coupling.

- $c > b$ with $H_0 > 0$,
- $c < b < 2c$ with $H_0 < 0$,
- $b > 2c$ with $H_0 < 0$,
- $c = b$ with $H_0 = 0$.

For $c > b$ case, we should have $H_0 > 0$, whereas in the $c < b < 2c$ case we have $H_0 < 0$. In both cases K_0 should be greater or equal to zero for the stability of the vacuum. In the third case, $b > 2c$ with $H_0 < 0$, for all the real values of K_0 , $a(t)$ diverges in the UV limit. This means that there is no chance for a nontrivial theory in that region. Finally the $c = b$ case with $H_0 = 0$ have already be shown in equation (22). It is clear that in the UV limits K_0 should not take negative values.

As seen above these constraints give different relations between numbers of color and flavor. Note that in all the cases studied, if we take $K_0 < 0$, one can deduce from equation (16) that $a(t)$ can be made equal to zero for a finite value of t , a situation which should not be allowed. Therefore, we can use only the option with $K_0 \geq 0$. The standard model with three colors and six flavors satisfies the $c > b$ case.

For $K_0 = 0$ at the UV limit, the equation (16)

$$a(t) = \frac{u(c-b)^2 e_0^2}{2H_0^2(2c-b)} \eta^{-1+\frac{2c}{b}}(t) \rightarrow +0, \quad (33)$$

shows that the coupling constant is asymptotically free. Also for a non zero K_0 , we find in the UV limit

$$a(t) \rightarrow \frac{(c-b)^2 e_0^2 K_0}{2H_0^2(2c-b)}. \quad (34)$$

Then the sign of the K_0 is crucial for the stability of the vacuum.

Although for $K_0 > 0$ we do not violate unitarity, we see that the asymptotic freedom criterion is not satisfied. The requirement of this criterion fixes K_0 at the value zero. In Figure 2, we plot the RG flows in $(a(t), g^2(t))$ plane for different values of H_0 higher than zero while the gauge coupling $\alpha(t=0)$ fixed to 1. The origin is the limit where t goes to infinity, there both coupling constants approach zero when $K_0 = 0$.

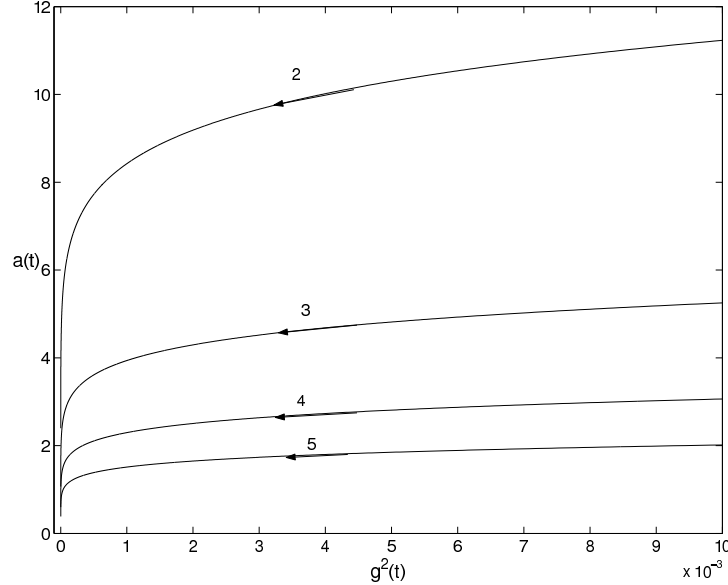


FIG. 2: Plot of $a(t)$ vs. $g^2(t)$ for different values of H_0 while $K_0 = 0$.

V. CONCLUSION

Here we write the $SU(N)$ gauge version of the polynomial lagrangian inspired by the Gürsey model. In [19] we had found an interacting model, where only the composites take part in scattering processes, if only perturbative calculations are done. Gauging it with a constituent $U(1)$ field resulted in a model which looked like the gauge-Higgs-Yukawa system, with all the problems associated with the Landau pole [21]. When a $SU(N)$ gauge field is coupled, instead, we find that the renormalization group equations for the three coupling constants indicate that this model is nontrivial. All the coupling constants go to zero asymptotically as the cut-off parameter goes to infinity, exhibiting the behavior dictated by asymptotic freedom.

In equations (24) and (25) we give the equations for the ratios of the coupling constants and find the fixed points. We see that we can have nontrivial fixed points.

One can apply the exact renormalization group to our model and obtain the additional vertices as given in our references [11] and [12]. This will be pursued in the future.

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